Mathematics: MCM 109 Final Exam Date: 20 – 1 – 2011 Duration Time: 1 Hour	Modern University For Technology & Information	Academic Year: 2010 – 2011 Semester: Autumn Examiner: Dr. Mohamed Eid
Answer 3 Questions Only	Faculty of Pharmacy	Marks

Questions	Marks
[1](a) Find y\` where: (i) $y = 4x^3 + 4^x$ (ii) $y = \cos x + \ln x$ (iii) $y = 3^x . \sin x$	3
(b) Find the integrals: (i) $\int (x^4 - 2^x) dx$ (ii) $\int (\frac{1}{x} + \cos x)\, dx$ (iii) $\int_0^1 [x^2 + 2]^2\, dx$	6
(c) Determine maximum and minimum values of the function $f(x) = x^3 - 3x$	3
[2](a) Find the integrals: (i) $\int x . \cos x\, dx$ (ii) $\int \frac{2x-1}{x^2 - 3x + 2}\, dx$	4
(b)If $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 0 & 2 \end{bmatrix}$. Find, if possible, $A + B$, $A + B^t$, $A.B$	4
(c)Solve the system of equations using inverse method: $x + y + z = 6$, $x - y + 2z = 4$, $2x - 2y + z = 2$	4
[3](a)Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$	6
(b)A drug in the blood decreases according to equation $\sqrt{y_0} - \sqrt{y} = 10t$. If the initial quantity $y_0 = 200$ units. Find (i)The time at which 30 % of drug exists in the blood. (ii)The time at which 50 % of drug exists in the blood. (iii)The time at which there is no drug in the blood.	6
[4](a)Find the limits: (i) $\lim_{x \to 3} \frac{x-3}{x^2 - 4}$ (ii) $\lim_{x \to 0} \frac{3^x - 2^x}{x}$ (iii) $\lim_{x \to 0} \frac{\ln(1+3x)}{x}$	6
(b)If a medicine is available in 3 dosage forms: First type of concentration: 1 mg /tablet Second type of concentration: 3 mg /tablet Third type of concentration: 4 mg /tablet If the pharmacist wanted to prepare 10 tablets containing 2 mg / tablet by mixing whole tablets of each type. Find all possible solutions.	6

Good luck *Dr. Mohamed Eid*

Answer

[1](a)(i) $y` = 12x^2 + 4^x.\ln 4$ (ii) $y` = -\sin x + \frac{1}{x}$

(iii) $y` = 3^x \cos x + 3^x.\ln 3.\sin x$

(b) (i) $\int (x^4 - 2^x)dx = \frac{x^5}{5} - \frac{2^x}{\ln 2} + c$ (ii) $\int (\frac{1}{x} + \cos x)\ dx = \ln x + \sin x + c$

(iii) $\int_0^1 [x^2+2]^2\ dx = \int_0^1 [x^4 + 4x^2 + 4]dx = \frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x = \frac{1}{5} + \frac{1}{3} + 4 = \frac{68}{15}$

(c) $f`(x) = 3x^2 - 3$, then $3x^2 - 3 = 0$. Then $x = 1$, $x = -1$

Since $f``(x) = 6x$, $f``(1) = 6$ and $f``(-1) = -6$

Then $x = 1$ is minimum and $x = -1$ is maximum

[2](a)(i) $\int (x \cos x)\ dx = x \sin x + \cos x + c$, integration by parts.

(ii) $\int \frac{2x\text{-}1}{x^2 - 3x + 2}\ dx = \int (\frac{3}{x-2} - \frac{1}{x-1}) = 3\ln(x-2) - \ln(x-1) + c$,

integration by partial fractions.

(b) A + B does not exist.

$$A + B^t = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 2 & 2 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ -3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -5 & 1 & 3 \\ -4 & -1 & 3 \\ 3 & 3 & -3 \end{bmatrix}$$

(c) The linear system written as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

$|A| = (-1 + 4) - (1 - 4) + (-2 + 2) = 6$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 3 & -3 & 3 \\ 3 & -1 & -1 \\ 0 & 4 & -2 \end{bmatrix}$$

Then the linear system has one solution:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3 & -3 & 3 \\ 3 & -1 & -1 \\ 0 & 4 & -2 \end{bmatrix}\begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ 12 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

[3](a) $|A-\lambda I| = \begin{vmatrix} 0-\lambda & 1 \\ 3 & 2-\lambda \end{vmatrix} = (-\lambda)(2-\lambda)-3=0$. Then $\lambda^2-2\lambda-3=0$

Then $\lambda=-1$, $\lambda=3$

For $\lambda=-1$, $\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}=0$. Then a + b = 0, 3a + 3b = 0 Or a = – b = any number

Put b = 1, then a = – 1. Then, the eigenvector $X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

For $\lambda=3$, $\begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}=0$. Then - 3a + b = 0, 3a - b = 0 Or 3a = b = any number

Put b = 3, then a = 1. Then, the eigenvector $X_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(b) Since $\sqrt{y_0}-\sqrt{y}=10t$. Then $\sqrt{200}-\sqrt{y}=10t$

(i) 30% = $\frac{30}{100}.200=60$ units.

Then $\sqrt{200}-\sqrt{60}=10t$ and time $t=\frac{\sqrt{200}-\sqrt{60}}{10}=0.64$ hour.

(ii) 50% = $\frac{50}{100}.200=100$ units.

Then $\sqrt{200}-\sqrt{100}=10t$ and time $t=\frac{\sqrt{200}-\sqrt{100}}{10}=0.4$ hour.

(iii) There is no drug in the blood when y = 0.

Then $\sqrt{200}=10t$ and time $t=1.4$ hour.

[4](a) (i) $\lim\limits_{x\to 3} \dfrac{x-3}{x^2-4} = \dfrac{3-3}{9-4} = 0$

(ii) $\lim\limits_{x\to 0} \dfrac{3^x - 2^x}{x} = \dfrac{0}{0} = \lim\limits_{x\to 0} 2^x \dfrac{(3/2)^x - 1}{x} = 2^0 \ln(3/2) = \ln(3/2)$

(iii) $\lim\limits_{x\to 0} \dfrac{\ln(1+3x)}{x} = 3$

(b) This problem can be formulated in mathematical model as:

Assume that x = number of tablets taken from the first type

y = number of tablets taken from the second type

z = number of tablets taken from the third type

Then $x + y + z = 10$, $x + 3y + 4z = 10*2 = 20$, $x, y, z \geq 0$, integers

It is a system of linear equations with integrality conditions.

This system can be solved as:

Rewrite this system as: $y + z = 10 - x$ (i)

$3y + 4z = 20 - x$ (ii)

Multiply equation (i) by – 4 and add to equation (ii) to eliminate z.

We get $-y = -40 + 4x + 20 - x$, then $y = 20 - 3x$.

From equation (i), $z = 10 - x - y = 10 - x - 20 + 3x = 2x - 10$

Then, the solution of the system is given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ 20-3x \\ 2x-10 \end{bmatrix}$$

Since the solution of the problem is positive integer.

Then, $x \geq 0$, $(20 - 3x) \geq 0$, $(2x - 10) \geq 0$, integers.

Then $5 \leq x \leq 20/3$, integer. Then $x = 5, 6$.

If $x = 5$, the solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$

If $x = 6$, the solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}$

Quiz I Date 23 – 3 – 2011 Name: .

(1) Find the following limits:

(a) $\lim\limits_{x\to 1}\dfrac{x^2-1}{x-2}$ (b) $\lim\limits_{x\to 1}\dfrac{x^{\frac{1}{2}}-1}{x^2-1}$ (c) $\lim\limits_{x\to 0}\dfrac{\log(1+3x)}{2x}$ (d) $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x^2+x^4}$

(2)Find y\` where: (a) $y = 2x^3 + 4^x$ (b) $y = \dfrac{2^x}{\sin x}$ (c) $y = 8 + \tan x.\ln x$

Answer

Quiz II Date 20 – 4 – 2011 Name: .

(1) Find the limits: (a) $\lim\limits_{x\to 1} \dfrac{x^2-2}{x+2}$ (b) $\lim\limits_{x\to 0} \dfrac{3^x-1}{3x}$ (c) $\lim\limits_{x\to 0} \dfrac{x}{x+\tan x}$

(2)Find y` where: (a) $y = x^3 \sin x$ (b) $y = 4^x + \log x$ (c) $y = 8 + x^{-3} + \tan x$

(3) Find the maximum and minimum values of the function: $f(x) = x + \dfrac{4}{x}$

(4) Find the integrals: (a) $\int\limits_0^1 (x^2+1)^2\, dx$ (b) $\int \dfrac{x+1}{x^2-3x+2}\, dx$

(c) $\int (\dfrac{1}{x^3} + \cos x)\, dx$ (d) $\int (x+1)\sin x\, dx$

Answer